UNIVERSITY OF SASKATCHEWAN
GE 124.3 – Engineering Mechanics I
FINAL EXAMINATION

TIME: 3 HOURS **December 17, 2003**

Only calculators, pens, pencils, and drawing aids are allowed for the exam.

Candidates are to answer all questions. All questions are of equal value.

You are to show your solution in the space below the question.

The reverse side of the page may be used if required.

NAME: SOLUTIONS
(First Name) **(Last Name)**

Section Number (Day/time): ____________________

Student Number: ____________________

Examination Room: ____________________

Marks

1. ________ **/10**
2. ________ **/10**
3. ________ **/10**
4. ________ **/10**
5. ________ **/10**
6. ________ **/10**
7. ________ **/10**
8. ________ **/10**

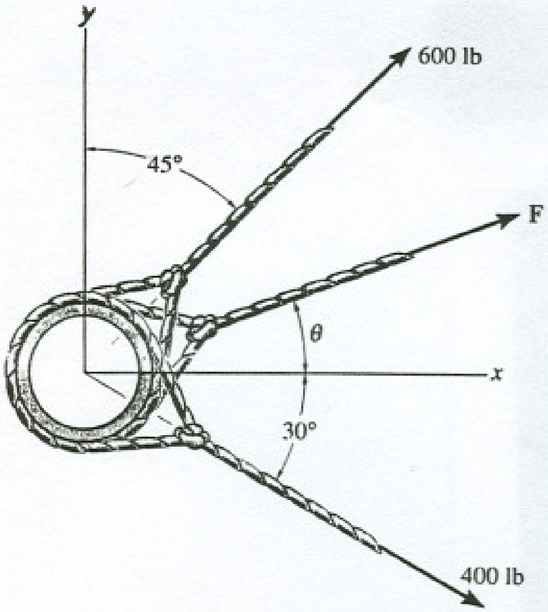
TOTAL: __________

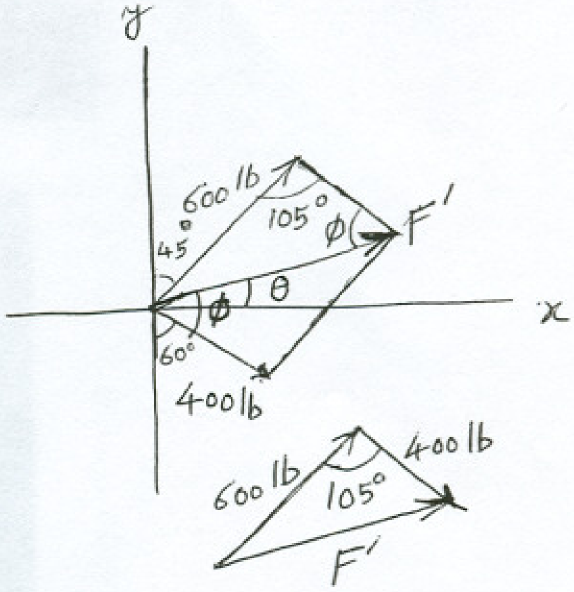
NOTE:
- make sure you have 8 problems in the exam
- use the space below each problem for your solution
- use the back of previous sheet if more room is required for your solution
- each question is worth the indicated marks
- please place your name at the top of each sheet
- a list of formulas is printed on the last sheet of this exam.

EXAM LOCATIONS:
Section 01 Education Gym

GE 124.3 Mechanics I - Final Examination 2003. Name: ______________________

1. Three cables pull on the pipe such that they create a resultant force of 900 lb magnitude. All forces lie in the x-y plane. Two of the cables are subjected to known forces of 600 lb and 400 lb as shown.
Determine the direction θ of the third cable so that the magnitude of force **F** in this cable is *minimum*. Find the magnitude of **F**. (*Hint*: First find the resultant of the two known forces.)





$$F' = \sqrt{(600)^2 + (400)^2 - 2(600)(400)\cos 105^\circ} = 802.64 \text{ lb}$$

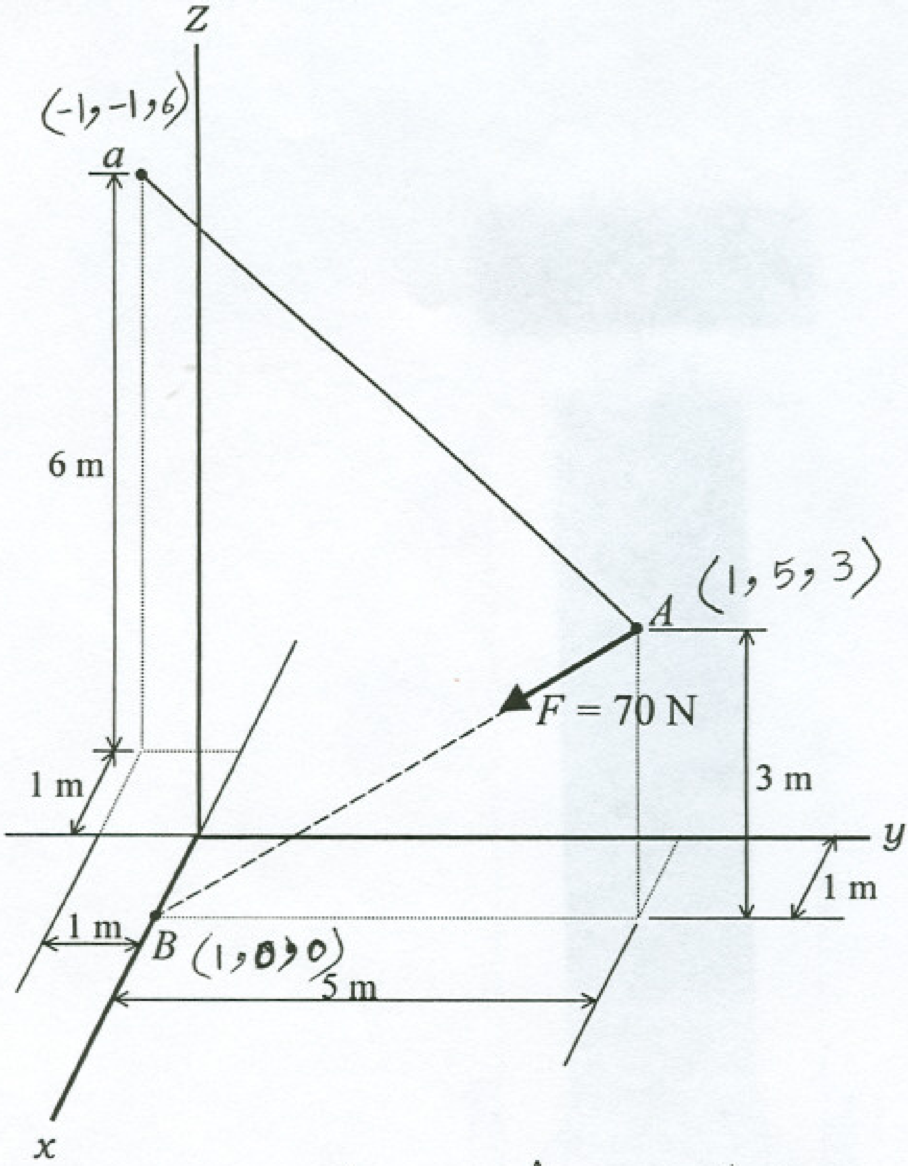
$$F = 900 - 802.64 = 97.36 \text{ lb} \Rightarrow \boxed{F' = 97.4 \text{ lb}}$$

$$\frac{\sin \phi}{600} = \frac{\sin 105^\circ}{802.64} \Rightarrow \phi = 46.22^\circ$$

$$\theta = 46.22^\circ - 30^\circ = 16.22^\circ \Rightarrow \boxed{\theta = 16.22^\circ}$$

GE 124.3 Mechanics I - Final Examination 2003. Name: ____________________

2. Determine the magnitudes of the projected components of the 70N force F_{AB} lying parallel to line *Aa* and lying perpendicular to the line *Aa*.



$$\vec{F} = 70\,N \cdot \frac{(1-1)\hat{i} + (0-5)\hat{j} + (0-3)\hat{k}}{\sqrt{0^2+(-5)^2+(-3)^2}} = \frac{70}{\sqrt{34}}\{-5\hat{j} - 3\hat{k}\} = \{-60.02\hat{j} - 36.01\hat{k}\}N$$

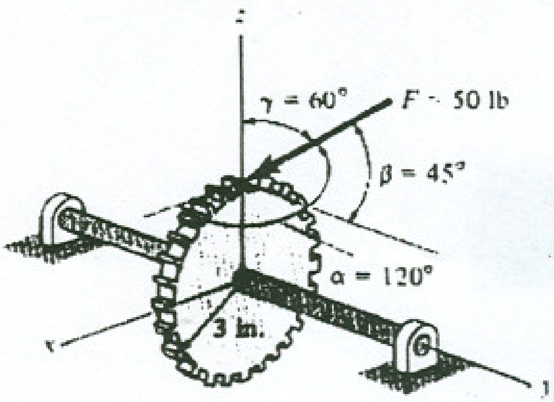
$$\hat{u}_{Aa} = \frac{(-1-1)\hat{i} + (-1-5)\hat{j} + (6-3)\hat{k}}{\sqrt{(-2)^2+(-6)^2+(3)^2}} = \frac{-2\hat{i} - 6\hat{j} + 3\hat{k}}{7}$$

$$F_{\parallel} = \vec{F} \cdot \hat{u}_{Aa} = (-60.02)(-\tfrac{6}{7}) + (-36.01)(\tfrac{3}{7}) = 36.01\,N$$

$$\boxed{F_{\parallel} = 36.0\,N}$$

$$F_{\perp} = \sqrt{F^2 - F_{\parallel}^2} = \sqrt{70^2 - (36.01)^2} = 60.0\,N \Rightarrow \boxed{F_{\perp} = 60.0\,N}$$

3. The 50-lb force acts on the 3-inch radius gear in the direction shown. The force direction cosine angles are $\alpha = 120°$, $\beta = 45°$ and $\gamma = 60°$. Determine the moment of this force about the y-axis of the shaft.



$M_y = \hat{j} \cdot (\vec{r} \times \vec{F})$

$\vec{r} = 3\hat{k}$ in.

$\vec{F} = -50\,lb\,\{\cos 120°\,\hat{i} + \cos 45°\,\hat{j} + \cos 60°\,\hat{k}\}$

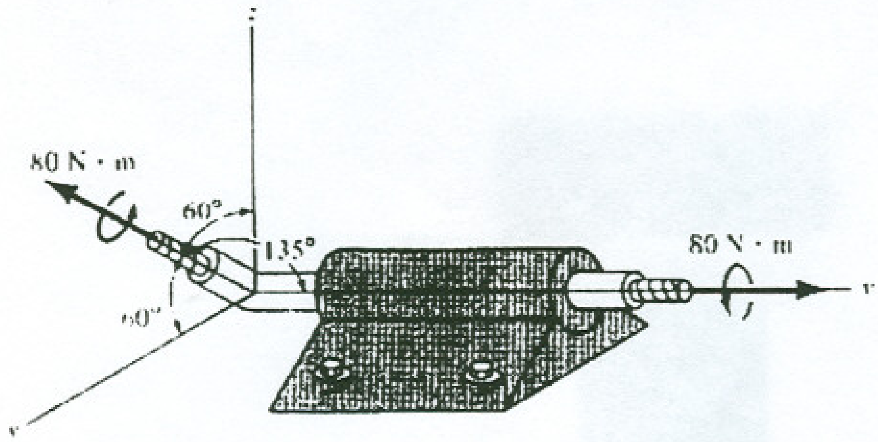
KN

$$M_y = \begin{vmatrix} 0 & 1 & 0 \\ 0 & 0 & 3 \\ -50\cos 120° & -50\cos 45° & -50\cos 60° \end{vmatrix}$$

$M_y = -(1)(-3)(-50\cos 120°) = +75$ in.lb

$\vec{M} = M_y\hat{j} = \boxed{75\hat{j} \text{ in.lb}}$

4. An electrical wire cable passes through and is held firmly by the support. The cable ends are subjected to the two 80 Nm couple moments shown. The direction cosine angles are $\alpha = 60°$, $\beta = 135°$ and $\gamma = 60°$ for the vector on the left side. Determine the resultant couple moment acting on the support specifying its magnitude and direction.



$$\vec{M} = \{(80\,Nm\,\cos 60°)\hat{i} + (80\,Nm\,\cos 135° + 80\,Nm)\hat{j} + (80\,Nm\,\cos 60°)\hat{k}\}$$

$$\vec{M} = \{40\hat{i} + 23.4\hat{j} + 40\hat{k}\}\ Nm$$

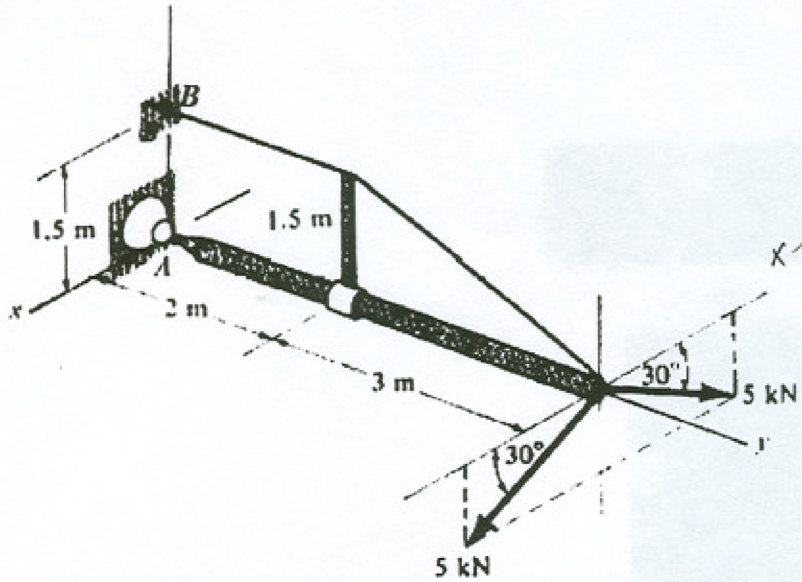
$$M = \sqrt{(40)^2 + (23.4)^2 + (40)^2} = 61.2\ Nm \Rightarrow \boxed{M = 61.2\ Nm}$$

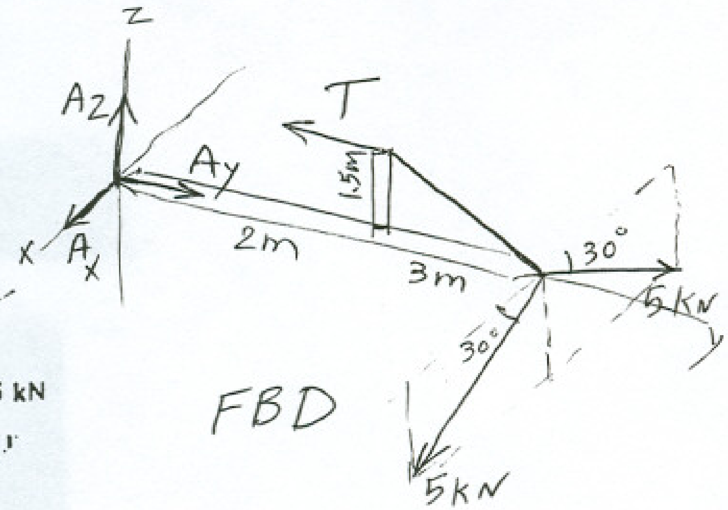
$$\alpha = \cos^{-1}\left(\frac{40\ Nm}{61.2\ Nm}\right) = 49.2° \Rightarrow \boxed{\alpha = 49.2°}$$

$$\beta = \cos^{-1}\left(\frac{23.4}{61.2}\right) = 67.5° \Rightarrow \boxed{\beta = 67.5°}$$

$$\gamma = \cos^{-1}\left(\frac{40}{61.2}\right) = 49.2° \Rightarrow \boxed{\gamma = 49.2°}$$

5. The boom is supported by a ball-and- socket joint at ***A*** and a guy wire at ***B***. If the loads in the two cables pulling on the end of the boom are each 5 kN and they lie in a plane which is parallel to the *x-z* plane, determine the components of reaction at A for equilibrium.





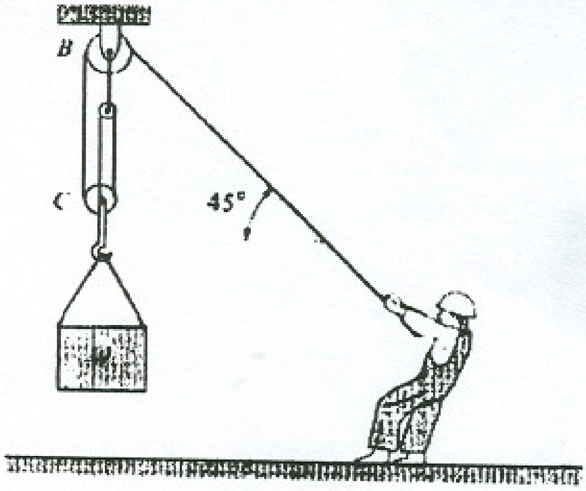
$$\Sigma M_x = 0 \Rightarrow T(1.5) - 2(5\sin 30^\circ)(5) = 0 \Rightarrow T = 16.67\,\text{KN}$$

$$\Sigma F_x = 0 \Rightarrow A_x + 5\cos 30^\circ - 5\cos 30^\circ = 0 \Rightarrow \boxed{A_x = 0\ \text{KN}}$$

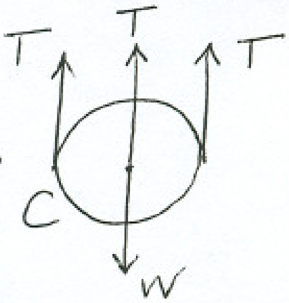
$$\Sigma F_y = 0 \Rightarrow A_y - T = 0 \Rightarrow \boxed{A_y = 16.67\,\text{KN}}$$

$$\Sigma F_z = 0 \Rightarrow A_z - 2(5\sin 30^\circ) = 0 \Rightarrow \boxed{A_z = 5\,\text{KN}}$$

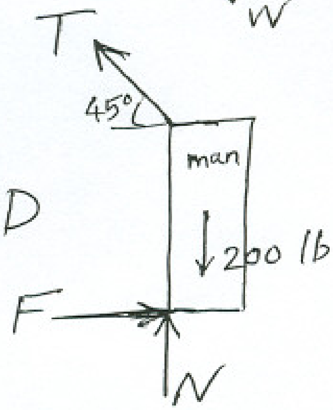
6. Determine the maximum weight W the man can lift using the pulley system. The man has a weight of 200 lb and the coefficient of friction between his feet and the ground is $\mu = 0.6$.



FBD



FBD



From FBD of pulley C: $T = \frac{1}{3} W$

From man's FBD:

$$\Sigma F_x = 0 \Rightarrow -T\cos 45^\circ + F = 0 \Rightarrow F = \frac{1}{3} W \cos 45^\circ$$

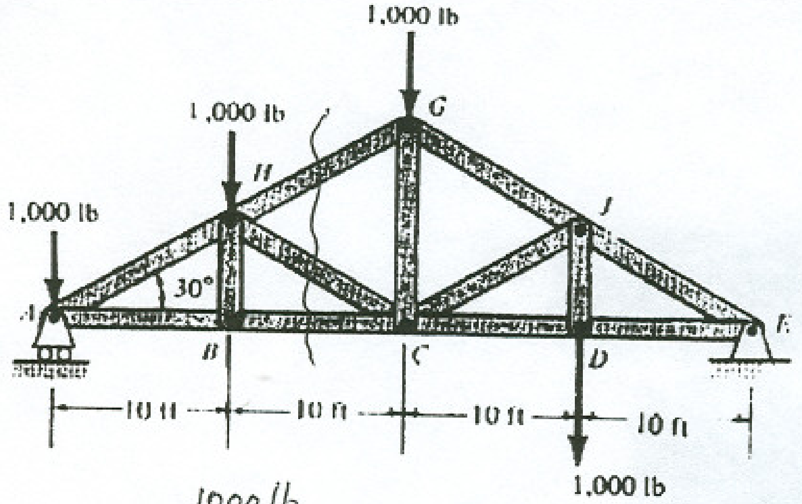
$$\Sigma F_y = 0 \Rightarrow T\sin 45^\circ - 200 + N = 0 \Rightarrow N = 200 - \frac{1}{3} W \sin 45^\circ$$

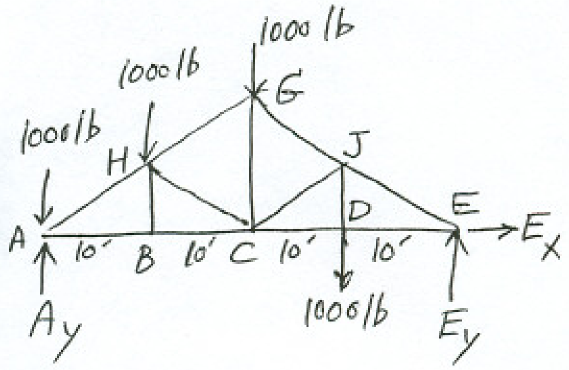
$$F = \mu N \Rightarrow \frac{1}{3} W \cos 45^\circ = 0.6\left(200 - \frac{1}{3} W \sin 45^\circ\right)$$

$$\therefore \boxed{W = 318 \text{ lb}}$$

GE 124.3 Mechanics I - Final Examination 2003. Name: ______________________

7. Determine the force in member BC of the pin connected truss. Indicate whether the member is in tension or compression.



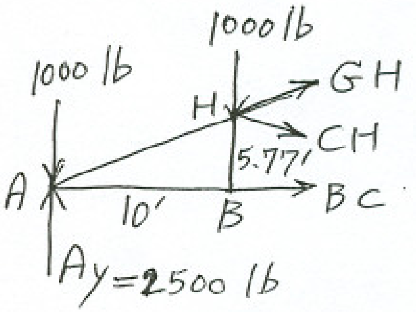


$\Sigma M_E = 0$

$-A_y(40) + 1000(40) + 1000(30)$

$+1000(20) + 1000(10) = 0$

$\Rightarrow A_y = 2500\ lb$

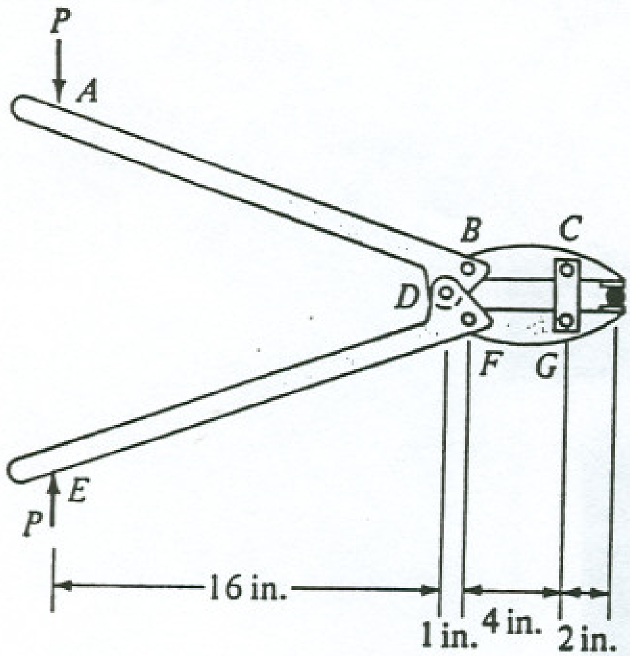


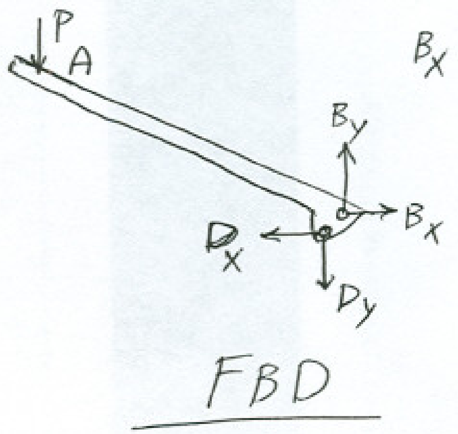
$$\Sigma M_H = 0 \Rightarrow BC(5.77) + 1000(10) - 2500(10) = 0$$

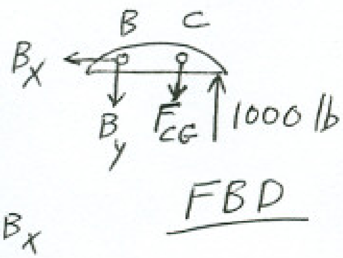
$$\Rightarrow \boxed{BC = 2600\ lb}$$

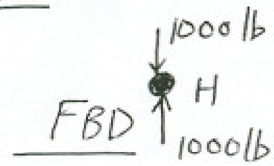
$$\boxed{BC = 2.60\ kip}$$

8. For the bolt cutter shown, determine the magnitude of the vertical forces P that must be applied at A and E to generate vertical forces of 1000 lb on the rod, H, to be cut. Also, determine the forces on each member of the machine under this loading condition.









Member BC: $\Sigma F_x = 0 \Rightarrow \boxed{B_x = 0}$

$\Sigma M_C = 0 \rightarrow 1000(2) + B_y(4) = 0 \Rightarrow \boxed{B_y = -500 \text{ lb}}$

$\Sigma F_y = 0 \Rightarrow -(-500) + 1000 - F_{CG} = 0 \Rightarrow \boxed{F_{CG} = 1500 \text{ lb}}$

Member AD:

$\Sigma F_x = 0 \Rightarrow \boxed{D_x = 0}$

$\Sigma M_D = 0 \Rightarrow P(16) + (-500)(1) = 0 \Rightarrow \boxed{P = 31.25 \text{ lb}}$

$\Sigma F_y = 0 \Rightarrow -31.25 - 500 - D_y = 0 \Rightarrow \boxed{D_y = -531.25 \text{ lb}}$

Because of symmetry, the forces on FG and DE are

Fundamental Equations of Statics

Cartesian Vector

$$\mathbf{A} = A_x\mathbf{i} + A_y\mathbf{j} + A_z\mathbf{k}$$

Magnitude

$$A = \sqrt{A_x^2 + A_y^2 + A_z^2}$$

Direction

$$\mathbf{u}_A = \frac{\mathbf{A}}{A} = \frac{A_x}{A}\mathbf{i} + \frac{A_y}{A}\mathbf{j} + \frac{A_z}{A}\mathbf{k}$$

$$= \cos\alpha\mathbf{i} + \cos\beta\mathbf{j} + \cos\gamma\mathbf{k}$$

$$\cos^2\alpha + \cos^2\beta + \cos^2\gamma = 1$$

Dot Product

$$\mathbf{A}\cdot\mathbf{B} = AB\cos\theta$$

$$= A_xB_x + A_yB_y + A_zB_z$$

Cross Product

$$\mathbf{C} = \mathbf{A}\times\mathbf{B} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ A_x & A_y & A_z \\ B_x & B_y & B_z \end{vmatrix}$$

Cartesian Position Vector

$$\mathbf{r} = (x_2 - x_1)\mathbf{i} + (y_2 - y_1)\mathbf{j} + (z_2 - z_1)\mathbf{k}$$

Cartesian Force Vector

$$\mathbf{F} = F\mathbf{u} = F\left(\frac{\mathbf{r}}{r}\right)$$

Moment of a Force

$$M_O = Fd$$

$$\mathbf{M}_O = \mathbf{r}\times\mathbf{F} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ r_x & r_y & r_z \\ F_x & F_y & F_z \end{vmatrix}$$

Moment of a Force About a Specified Axis

$$M_a = \mathbf{u}\cdot\mathbf{r}\times\mathbf{F} = \begin{vmatrix} u_x & u_y & u_z \\ r_x & r_y & r_z \\ F_x & F_y & F_z \end{vmatrix}$$

Simplification of a Force and Couple System

$$\mathbf{F}_R = \Sigma\mathbf{F}$$

$$(\mathbf{M}_R)_O = \Sigma\mathbf{M}_O$$

Equilibrium

Particle

$$\Sigma F_x = 0,\ \Sigma F_y = 0,\ \Sigma F_z = 0$$

Rigid Body-Two Dimensions

$$\Sigma F_x = 0,\ \Sigma F_y = 0,\ \Sigma M_O = 0$$

Rigid Body-Three Dimensions

$$\Sigma F_x = 0,\ \Sigma F_y = 0,\ \Sigma F_z = 0$$

$$\Sigma M_{x'} = 0,\ \Sigma M_{y'} = 0,\ \Sigma M_{z'} = 0$$

Friction

Static (maximum) $F_s = \mu_s N$

Kinetic $F_k = \mu_k N$

Center of Gravity

Particles or Discrete Parts

$$\bar{r} = \frac{\Sigma\tilde{r}W}{\Sigma W}$$

Body

$$\bar{r} = \frac{\int \tilde{r}\,dW}{\int dW}$$

Area and Mass Moments of Inertia

$$I = \int r^2\,dA \qquad I = \int r^2\,dm$$

Parallel-Axis Theorem

$$I = \bar{I} + Ad^2 \qquad I = \bar{I} + md^2$$

Radius of Gyration

$$k = \sqrt{\frac{I}{A}} \qquad k = \sqrt{\frac{I}{m}}$$

Virtual Work

$$\delta U = 0$$